Warm-up!

A) Solve over the interval $0 \leq \theta < 2\pi$:
$6\sin\theta + 2 = -1$

B) Solve over the interval $0 \leq \theta < 2\pi$:
$\tan^2\theta - \sec\theta - 1 = 0$

C) I'm thinking of a number. When squared, my number is 25. Of what number am I thinking?

Periodic Graphs Day 7:

- (3.9) Inverse Trigonometric Functions
- (3.10) Solving Trig Equations
- Solving Inverse Trig Equations

Homework: Inverse Trig Functions

Quiz The Class After Next:

- An FRQ #3 style question
- Graphs of tan/cot/sec/csc
- Solving Trig Equations Using Inverse Trig

$\arcsin(x)$

$\sqrt{25}$ asks the question "What positive number makes 25 when you square it?"
In other words, $x^2 = 25$ has two solutions, but $\sqrt{x}$ is only asking for the solution in $[0, \infty)$

$\arcsin\left(\frac{1}{2}\right)$ asks the question "What angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ makes $\frac{1}{2}$ when you take the sine?"
In other words, $\sin\theta = \frac{1}{2}$ has an infinite number of solutions, but $\arcsin\left(\frac{1}{2}\right)$ is only asking for the solution in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Practice:

$\arcsin\left(-\frac{1}{2}\right) =$ $\arcsin\left(\frac{\sqrt{3}}{2}\right) =$ $\arcsin(-1) =$

$\arctan(x)$

$\arctan\left(\sqrt{3}\right)$ asks the question "What angle in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ makes $\sqrt{3}$ when you take the tangent?" In other words, $\tan\theta = \sqrt{3}$ has an infinite number of solutions, but $\arctan\left(\sqrt{3}\right)$ is only asking for the solution in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Practice:

$\arctan(1) =$ $\arctan\left(-\frac{\sqrt{3}}{3}\right) =$ $\arctan(0) =$

$\arccos(x)$

$\arccos\left(-\frac{\sqrt{3}}{2}\right)$ asks the question "What angle in $[0, \pi]$ makes $-\frac{\sqrt{3}}{2}$ when you take the cosine?"

In other words, $\cos\theta = -\frac{\sqrt{3}}{2}$ has an infinite number of solutions, but $\arccos\left(-\frac{\sqrt{3}}{2}\right)$ is only asking for the solution in $[0, \pi]$

Practice:

$\arccos(1) =$ $\arccos\left(-\frac{\sqrt{2}}{2}\right) =$ $\arccos\left(-\frac{1}{2}\right) =$

Some things to note

The range of each of these three inverse trig functions is different. Each includes the first quadrant and a neighboring quadrant.

$\arcsin(x)$ Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\arctan(x)$ Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\arccos(x)$ Range: $[0, \pi]$

Sometimes $\arcsin(x)$ is written as $\sin^{-1}(x)$
Sometimes $\arctan(x)$ is written as $\tan^{-1}(x)$
Sometimes $\arccos(x)$ is written as $\cos^{-1}(x)$

Trig functions turn angles into ratios. Inverse trig functions turn ratios back into angles.

Find the angle for each inverse function (without a calculator)

A) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

C) $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

Game version of this:

B) $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$

D) $\arctan(-1)$

Find the angle for each inverse function (without a calculator)

E) $\arccos\left(-\frac{1}{2}\right)$

G) $\sin^{-1}(1)$

Game version of this:

F) $\arccos(0)$

H) $\arctan\left(-\sqrt{3}\right)$

Solve

$$3\tan\theta + 5 = 1$$

Which of the following are solutions?

A) $\arctan\left(\frac{4}{3}\right)$

B) $\pi - \arctan\left(\frac{4}{3}\right)$

C) $\pi + \arctan\left(\frac{4}{3}\right)$

D) $2\pi - \arctan\left(\frac{4}{3}\right)$

E) $2\pi + \arctan\left(\frac{4}{3}\right)$

F) $\arctan\left(-\frac{4}{3}\right)$

Solve

$$5\sin\theta + 6 = 2$$

Which of the following are solutions?

A) $\sin^{-1}\left(\frac{4}{5}\right)$

B) $\sin^{-1}\left(-\frac{4}{5}\right)$

C) $2\pi - \sin^{-1}\left(\frac{4}{5}\right)$

D) $\pi - \sin^{-1}\left(\frac{4}{5}\right)$

E) $\pi + \sin^{-1}\left(\frac{4}{5}\right)$

Solve over the interval $0 \leq \theta < 2\pi$. Write unfamililar angles in terms of arccosine.

$$6\cos^2\theta + \cos\theta - 1 = 0$$

Find ALL solutions. Write unfamiliar solutions in terms of arccosine.

$$\sec^2\theta - 4\sec\theta - 5 = 0$$

Step 1: Isolate the inverse trig function

Step 2: Apply the corresponding trig function

$$2\arctan(3x-1)=\frac{\pi}{2}$$

Inverse Trig Equations

Let $p(x) = 3\sin^{-1}(\pi x) + \pi$

Find all input values in the domain of p that yield an output value of $\frac{3\pi}{2}$

Inverse Trig Equations

Let $r(x) = \cos^{-1}(6x)$

Solve $r(x) = \frac{\pi}{3}$ for all values of x in the domain of r.

More Evaluating Practice

$\arctan(-1)$

$\arcsin\left(-\frac{\sqrt{2}}{2}\right)$

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\cos^{-1}\left(-\frac{1}{2}\right)$

$\arccos(0)$

$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$